QMaxSATpb: A Certified MaxSAT-Solver

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ARTIFICIAL
INTELLIGENCE
RESEARCH GROUP

Thanks to Jakob Nordström for sharing his proof logging slides.

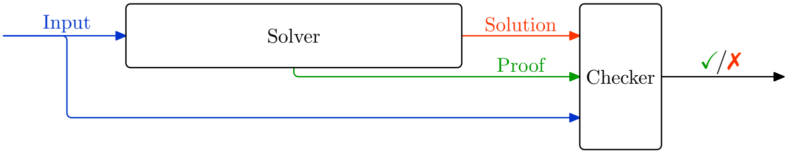
COMBINATORIAL SOLVING AND OPTIMISATION

- Revolution last couple of decades in combinatorial solvers for
  - Boolean satisfiability (SAT) solving [BHvMW21]
  - Satisfiability modulo theories (SMT) solving [BHvMW21]
  - Constraint programming (CP) [RvBW06]
  - Mixed integer linear programming (MIP) [AW13, BR07]
  - Answer Set Programming (ASP) [GKKS12]
- Solve NP problems (or worse) very successfully in practice!
- Except solvers are sometimes wrong... [BLB10, CKSW13, AGJ+18, GSD19, GS19]
- Software testing doesn't suffice to resolve this problem
- Formal verification techniques cannot deal with level of complexity of modern solvers

CERTIFIED RESULTS WITH PROOF LOGGING

Design certifying algorithms [ABM+11, MMNS11] that

- not only solve problem but also
- do proof logging to certify that solution is correct



Workflow:

1. Run solver on problem input
2. Get as output not only solution but also proof
3. Feed input + solution + proof to proof checker
4. Verify that proof checker says solution is correct

YET ANOTHER SAT SUCCESS STORY

Well established — required in main track of SAT competitions

Many proof logging formats for SAT solving using CNF clausal format:

- *DRAT* [HHW13a, HHW13b, WHH14]
- *GRIT* [CMS17]
- *LRAT* [CHH+17]
- . . .

Formally verified proof verifiers exist.

But efficient proof logging has remained out of reach for other paradigms,
e.g. Maximum Satisfiability (MaxSAT)

OUTLINE OF THIS PRESENTATION

The rest of this presentation:

- MaxSAT solver *QMaxSAT* [KZFH12]
- *VeriPB* [BGMN22, EGMN20b] as proof system.
- Our contribution: *QMaxSATpb*, A certified MaxSAT solver, by example
- Experimental results
- Future work & Conclusions

PARTIAL MAXSAT

A partial MaxSAT-instance is a tuple (F, S) with:

- F the set of hard clauses.
- S the set of soft clauses.

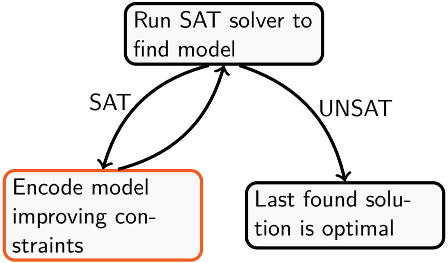
A solution is an assignment for all variables such that:

- All hard clauses are satisfied.
- No other satisfying assignment satisfies more soft clauses.

QMaxSAT: IDEA BEHIND THE SOLVER

QMaxSAT [KZFH12] is an Iterative Satisfiability-Based MaxSAT solver.

- Given a satisfying assignment, QMaxSAT searches for another assignment with fewer soft clauses falsified.
- Totalizer encoding of cardinality constraints [BB03]



VeriPB: A GENERAL PURPOSE PROOF SYSTEM

VeriPB is a proof system for pseudo-Boolean optimisation [BGMN22, EGMN20b].

It reasons on 0–1 integer linear inequalities $\sum_i a_i l_i \geq A$ (a.k.a. pseudo-Boolean constraints) with:

- Cutting Planes (CP) proof system [CCT87]
  - e.g., adding up two constraints
- Reverse Unit Propagation [GN03]
  - allows deriving constraints without providing an explicit derivation
- Redundance-Based Strenghtening [GN21, BGMN22]
  - generalisation of the RAT-rule [BT19]
  - allows introducing "fresh" reification variables, such as $r \Leftrightarrow (\sum_i a_i l_i \geq A)$.
- Support for Optimisation [BGMN22]
  - allows deriving model-improving constraints

QMaxSATpb: AN EXAMPLE

Hard Clauses	Soft Clauses
$\overline{x}_1 \vee x_2$	$\overline{x}_1 \vee \overline{x}_2 \vee r_1$
$x_1 \vee \overline{x}_2$	$x_1 \vee x_2 \vee r_2$
$\overline{x}_2 \vee x_3$	$x_2 \vee x_4 \vee r_3$
$\overline{x}_3 \vee x_4$	

- Relaxation variables r_i such that C_i falsified implies r_i true.
- We want to minimize $\sum_i r_i$

QMaxSATpb BY EXAMPLE

Objective: *min* $\sum_i r_i$

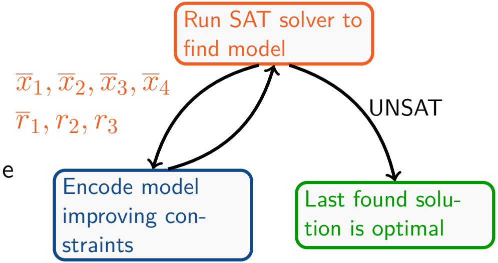
VeriPB proof:

derived	justification
$x_2 + r_2 \geq 1$	Reverse Unit Propagation
$\{\overline{x}_1, \ldots, \overline{x}_4, \overline{r}_1, r_2, r_3\}$	Incumbent solution
$\sum_i r_i \leq 1$	Objective Improvement Rule
$\mathrm{PB}(p_1 \Leftrightarrow (\sum_i r_i \geq 1))$	Fresh variable (RBS)
$\mathrm{PB}(p_2 \Leftrightarrow (\sum_i r_i \geq 2))$	
$\mathrm{CNF}(p_j \Leftrightarrow (\sum_i r_i \geq j))$	Explicit CP derivation
$\overline{p}_2 \geq 1$	Explicit CP derivation
$x_4 \geq 1$	Reverse Unit Propagation
$\{\overline{x}_1, \overline{x}_2, \overline{x}_3, x_4, \overline{r}_1, r_2, \overline{r}_3\}$	Incumbent solution
$\sum_i r_i \leq 0$	Objective Improvement Rule
$\overline{p}_1 \geq 1$	Explicit CP derivation
$0 \geq 1$	Reverse Unit Propagation

$\overline{x}_1 \vee x_2$	$\overline{x}_1 \vee \overline{x}_2 \vee r_1$
$x_1 \vee \overline{x}_2$	$x_1 \vee x_2 \vee r_2$
$\overline{x}_2 \vee x_3$	$x_2 \vee x_4 \vee r_3$
$\overline{x}_3 \vee x_4$	$x_2 \vee r_2$

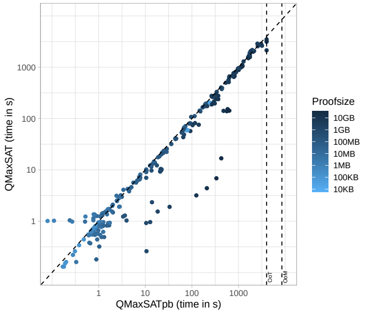
$\mathrm{CNF}(p_j \Leftrightarrow (\sum_i r_i \geq j))$

$\overline{p}_2$	x_4
$\overline{p}_1$	$\perp$

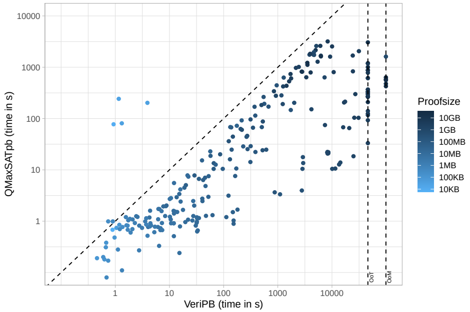


EXPERIMENTAL RESULTS

MaxSAT evaluations 2021
Resource Limits: *QMaxSAT* (1h, 32GB) — *VeriPB* (10h, 64GB)
10.2% OoT, 2.4% OoM



(a) Performance overhead of proof logging



(b) Performance of proof verification

FUTURE RESEARCH DIRECTIONS

Other MaxSAT Algorithms

- Iterative MaxSAT algorithms (e.g., Pacose [PRB18])
  - Uses different encodings of cardinality constraints
- Core-guided MaxSAT algorithms [Ave21, IMMS19, JA17, MTNJ+17]
  - Heavily rely on encodings of cardinality constraints
- Implicit hitting sets solvers [DB11]
  - Challenge: certifying the minimality of the hitting sets.

Proof logging for other combinatorial optimization

- Pseudo-Boolean optimization
- Mixed integer linear programming *(work on SCIP in [CGS17, EG21])*
- Satisfiability modulo theories (SMT) solving *(work by Bjørner and others)*
- Answer Set Programming *(ASP–DRUPE [ADF+19], but no (native) support for optimization or aggregates (PB constraints). ASP-VeriPB(?))*

SUMMING UP

Proof logging helps:

- Ensuring correctness of a result.
- Debugging in case of a bug.
- Building trust in solvers

VeriPB: general-purpose proof system

- Subgraph Isomorphism Problem [GMN20]
- Parity (XOR) reasoning [GN21]
- All Different reasoning (CP) [EGMN20a]

QMaxSATpb: Certified MaxSAT-solver

- Clauses derived by SAT oracle are RUP
- Totalizer encoding of cardinality constraints can be proven by an explicit CP derivation
- Proof logging is possible without too much overhead, verifying proofs is harder.

Thank you for your attention!

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0–1 INTEGER LINEAR (A.K.A. PSEUDO-BOOLEAN) CONSTRAINTS

Pseudo-Boolean (PB) constraints are 0–1 integer linear constraints

$$C \doteq \sum_i a_i \ell_i \geq A$$

- $a_i, A \in \mathbb{Z}$
- literals ℓ_i: x_i or $\overline{x}_i$ (where $x_i + \overline{x}_i = 1$)
- variables x_i take values $0 = \mathit{false}$ or $1 = \mathit{true}$

Pseudo-Boolean formulas are conjunctions of pseudo-Boolean constraints

A pseudo-Boolean optimisation problem is a formula F with a linear objective function.

SOME TYPES OF PSEUDO-BOOLEAN CONSTRAINTS

1. Clauses

$$x \vee \overline{y} \vee z \quad \Leftrightarrow \quad x + \overline{y} + z \geq 1$$

2. Cardinality constraints

$$x_1 + x_2 + x_3 + x_4 \geq 2$$

3. General pseudo-Boolean constraints

$$x_1 + 2\overline{x}_2 + 3x_3 + 4\overline{x}_4 + 5x_5 \geq 7$$

PSEUDO-BOOLEAN REASONING: CUTTING PLANES [CCT87]

Literal axioms $$\frac{}{\ell_i \geq 0}$$

Linear combination $$\frac{\sum_i a_i \ell_i \geq A \qquad \sum_i b_i \ell_i \geq B}{\sum_i (c_A a_i + c_B b_i) \ell_i \geq c_A A + c_B B} \quad [c_A, c_B \in \mathbb{N}]$$

Division $$\frac{\sum_i c a_i \ell_i \geq A}{\sum_i a_i \ell_i \geq \lceil A/c \rceil} \quad [c \in \mathbb{N}^+]$$

Toy example:

$$\dfrac{\text{Lin comb}\ \dfrac{2x + 4y + 2z + w \geq 5 \qquad 2x + y + w \geq 2}{6x + 6y + 2z + 3w \geq 9} \qquad \overline{z} \geq 0}{\text{Div}\ \dfrac{6x + 6y + 3w \geq 7}{2x + 2y + w \geq 3}}\ \text{Lin comb}$$

EXTENSION RULE: REDUNDANCE-BASED STRENGTHENING

C is redundant with respect to F if F and $F \wedge C$ are equisatisfiable

Want to allow adding redundant constraints

Redundance-based strengthening [BT19, GN21, BGMN22]

C is redundant with respect to F if and only if there is a substitution ω (mapping variables to truth values or literals), called a witness, for which

$$F \wedge \overline{C} \models (F \wedge C)\restriction_{\omega}$$